GRAVITATIONAL WAVES CARRYING ORBITAL ANGULAR MOMENTUM

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Binary black holes in orbit loose energy thereby coming close to each other and ultimately merge. This process also radiates large amount of orbital angular momentum. Thus gravitational waves (GWs) not only carry energy but also orbital angular momentum.

$$\frac{dL^i}{dt} = \frac{c^3}{32\pi G}\epsilon^{ijk}\int d\Omega r^2 \langle \dot{h}_{ab} x^j \partial^k h_{ab}\rangle$$
$$= \frac{2G}{15c^5}\epsilon^{ijk}\langle \ddot{Q}_{ja}\dddot{Q}_{ka}\rangle$$
$$\simeq \frac{2G}{15c^5}(\mu r^2)^2\omega^5$$

$$\frac{dL^i}{dt} \sim 10^{34} \text{ J}$$

$\mu \sim 30\ M_\odot$
r ~ 200 km
$\omega \sim 10^5$ Hz

Had the Earth lost orbital angular momentum at this rate; it would collide with the sun in 1 ms !!

#KEY IDEA 1

Compact object binaries loose angular momentum along with energy.

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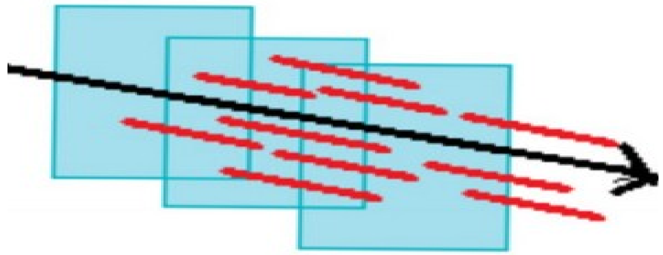
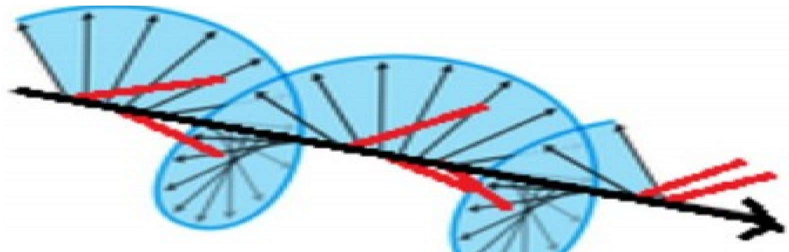
In plane waves, the direction of momentum transfer (direction of Poynting vector) is parallel to the direction of propagation and thus carry no angular momentum. More specifically the total orbital angular momentum is proportional to $\int (x^i k^j - x^j k^i)\, d^3x$ which is zero as d^3x is a rotationally invariant measure while the others are three vector components. Thus in other words, the plane wave approximation throws away information about angular momentum.



Plane Wave

This problem can be resolved by looking for solutions of the linearized Einstein's equations which are not plane. **To carry angular momentum a radiation has to have a position dependent polarisation tensor** which is equivalent to having a spacetime dependent phase term.



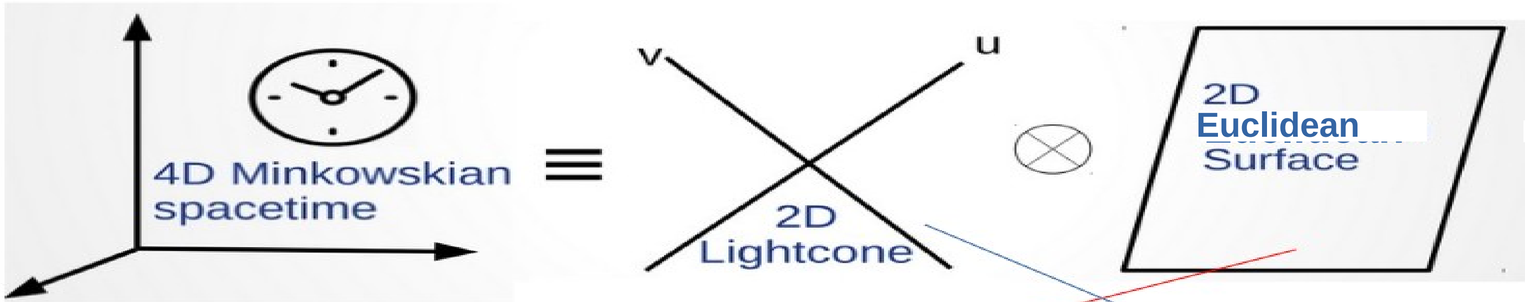
Beam with OAM

BLACK: Dirn. of Propagation
RED: Dirn. of momentum
BLUE: Polarisation tensor

#KEY IDEA 2

Plane wave approximation throws away information regarding angular momentum radiated.

- GWs propagate with the speed of light (null) on 4D flat (Minkowskian) spacetime and oscillations are perpendicular to the direction of propagation (transverse).
- 4D Minkowskian spacetime can be written as a product of a 2D light cone and a 2D flat (Euclidean) surface.
- The polarization which lies on the Euclidean Surface has to have non-trivial spatial dependence for such a transverse wave to carry angular momentum.



$$h^{(1)\alpha\beta} = \sum_{m=0}^{\infty} A_m(\kappa, C)^{\alpha\beta} J_m(\kappa\rho) \exp(im\phi) \exp(iCv + i\frac{\kappa^2}{C}u)$$

We refer to these solutions with spacetime dependent polarisation as gravitational wave beams.

$h^{(1)\alpha\beta}$ are the perturbations over flat spacetime in linear order.

This solutions can be iterated to higher orders!!

#KEY IDEA 3

Gravitational wave beams are essential and carries information regarding radiated OAM.

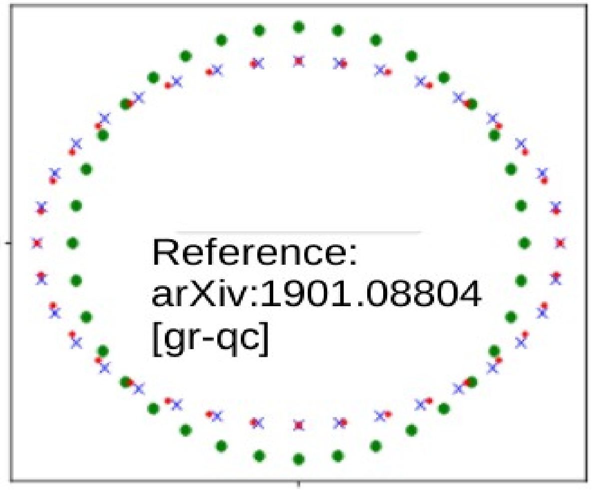
Detection in an interferometer

GWs stretch and contract spacetime thus creating a strain which can be detected by a Michelson-like interferometer.

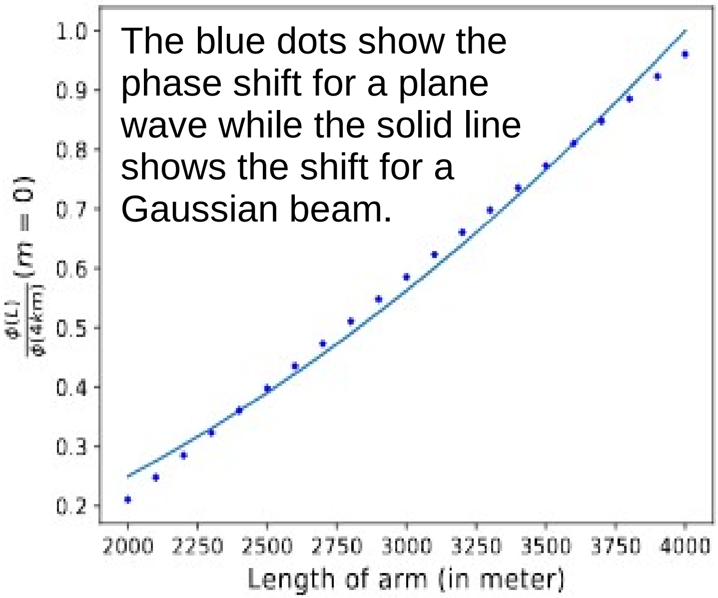
For plane waves strain induced due to gravitational wave is constant.

If we consider a GW beam strain is not constant or is a function of coordinates.

What this means is a circular ring of particles which changes to an ellipse upon the passage of plane waves gets distorted.



The green dots represents particles in absence of GWs. The red dots are the position of the same in presence of a plane wave. The blue crosses are the positions due to a passage of a GW beam.



The blue dots show the phase shift for a plane wave while the solid line shows the shift for a Gaussian beam.

Conclusion

Gravitational waves should have a spacetime dependent polarization. Detection of this effect is crucial for a direct measurement of angular momentum. A direct measurement of angular momentum shall give a new handle to constrain the source. The specifications of instruments required to measure such effects is yet to be studied.